Problem 5

CHANGE(n, V, D)

FOR v = 0 TO V

M[0, v] ← 0

FOR i = 1 TO n

FOR v = 0 TO V

IF (Di > v)

M[i, v] ← M[i – 1, v].

ELSE

M[i, v] ← min { M[i – 1, v], 1 + M[i – 1, v – Di] }.

combinations = []

start = V

i = n

WHILE (start != 0 )

IF M[i, start] == M[i-1, start]

i = i – 1

ELSE

combinations 🡨 Di

start = start – Di

return combinations

Discussing optimality:

Assistance taken from slides

M[i, v] = optimal number of coins with denomination 1,….i subjected to limit V

Goal = M[i, V]

Case 1: item i is not selected then the best of {1,2,3…, i-1} are selected subject to limit V

Case 2 : item i selected then,

V = V – Di

number of coins ++

then the best of {1,2,3…, i-1} are selected subject to limit V

So due to exchange argument, these substructures are optimal, hence, making the whole solution optimal.